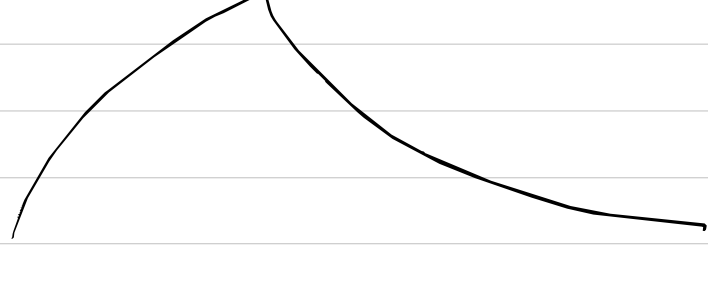


Week 7 Out of lab  
 (13/2)  
 22:00



I made an RC circuit in the lab.

In an RC circuit when I apply a step input, the capacitor voltage  $V(t)$  rises gradually according to

$$\frac{dV}{dt} = \frac{V_0 - V(t)}{RC}$$

Why is this?

Eq 3 -  $Q = CV$

Eq 4 -  $\frac{dQ}{dt} = I(t) = C \frac{dV}{dt}$

$$V_0 = V_R + V(t) = RC \frac{dV}{dt} + V(t)$$

$$V_0 - V(t) = RC \frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{V_0 - V(t)}{RC}$$

when  $V(0) = 0$ ,

$$\frac{dV}{dt} = \frac{V_0 - V}{RC}$$

$$\int_0^{V(t)} \frac{dV'}{V_0 - V'} = \int_0^t \frac{dt'}{RC}$$

$$-\ln[V_0 - V(t)] - (-\ln[V_0]) = \frac{t}{RC}$$

$$-\ln[V_0 - V(t)] + \ln[V_0] = \frac{t}{RC}$$

$$-\ln[V_0 - V(t)] + 1 = \frac{t}{RC}$$

$$\Rightarrow V(t) = V_0 (1 - e^{-t/RC})$$

$$\tau_{step} = RC$$

For natural response,

$$\frac{dV}{dt} = -\frac{V}{RC}$$

and moving along w/ what I did before,

$$V(t) = V_0 e^{-t/RC}$$

$$\tau = RC$$

$$\tau = 100 \times 10^3 \times 560 \times 10^{-12} \approx 56 \mu s$$

I measured  $58 \mu s$  which is  $2 \mu s$  away

$$V_{in} - V(t) = V_0 e^{-t/RC}$$

$$\ln(V_{in} - V(t)) = \ln(V_0) - \frac{t}{RC}$$

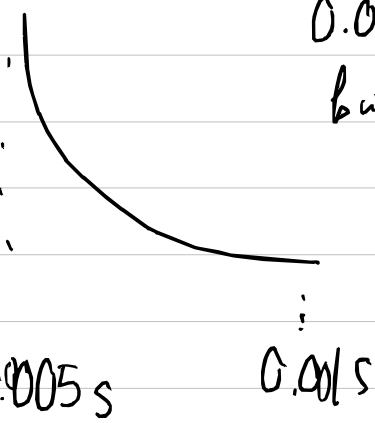
If I plot  $\ln(V_{in} - V(t))$  vs time  $t$ , I should have a straight line w/ slope  $-1/RC = -1/\tau$ .  $\tau = 56 \mu s$

$$-\frac{1}{56 \times 10^{-6}} = -17857$$

Go to Figure 1 in my python code

My plot has a  $\tau$  of  $67 \mu s$  and a slope of  $-14800$ . I don't know why.

I tried changing the time frame from



$0.0005s \sim 0.0006s$  to  $0.005s \sim 0.01s$  but it gave me a worse value.

This plot was based off from my CSV file.

Now let me plot using the data points documented in lab (w/o csv file). Hopefully this one will give us  $56 \mu s$  and a slope of  $-17857$ .

Figure 2. I am happy with this plot.

I had to adjust the parameters in the line to when the step and natural responses begin, otherwise the equation doesn't fit and I would get wrong numbers.

If I compare figure 2 to what I did in lab, the model describes my observations well. I am still wondering why I have a result of  $2 \mu s$  off, but this may be due to the resistor's tolerance ( $\pm 5\%$ ) and the capacitor's tolerance too. I might have to measure the exact capacitance and resistance in the future to increase my precision for expected  $\tau$  values.

On the next page, I will describe what I did for week 8's out of lab.

3/3 01:00 end

3/4 10:00 AM

I plugged in the csv file into a plot.

Figure 4 RLC  $\beta = \frac{R}{2L}$   $\omega_0^2 = \frac{1}{LC}$

$$\ddot{U}_c + 2\beta \dot{U}_c + \omega_0^2 U_c = 0$$

$$U_c(t) = U_0 e^{i\omega t}$$

$$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

Solution in general

$$U_c(t) = e^{-\beta t} (U_0^{(1)} e^{i\omega t} + U_0^{(2)} e^{-i\omega t})$$

initial condition

$$U_0 = U_c(0) \quad \dot{U}_c(0) = 0$$

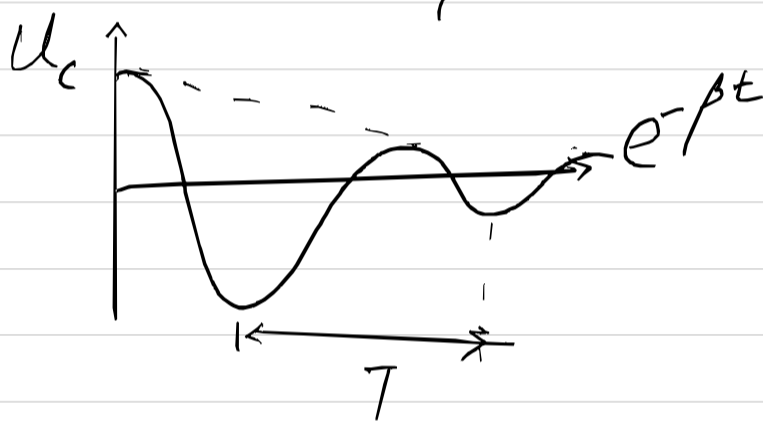
$$U_0^{(1)} = \frac{1}{2} U_0 (1 - i \frac{\beta}{\omega})$$

$$U_0^{(2)} = \frac{1}{2} U_0 (1 + i \frac{\beta}{\omega})$$

$$U_c(t) = U_0 e^{-\beta t} (\cos(\omega t) + \frac{\beta}{\omega} \sin(\omega t))$$

fully charged  
Current

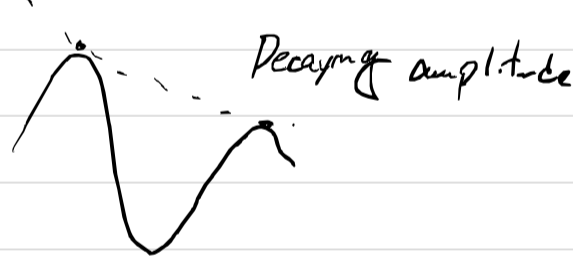
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}}$$



$$I = C \dot{U}_c$$

$$= \frac{U_0}{\omega L} e^{-\beta t} \sin(\omega t + \pi)$$

The decaying is like a damped oscillator, so I will find the  $T$  in this case.



$$T = \frac{t_2 - t_1}{\ln\left(\frac{V_1}{V_2}\right)} \approx 7.22 \mu s$$

$$V(t) = V_0 e^{-\frac{t}{\tau}}$$

Figure 5

$$\ln(V(t)) = \ln(V_0) - \frac{t}{\tau}$$

$$\text{slope: } -\frac{1}{\tau}$$

If the decay is exponential, then the  $\ln$  value is linear.

I was happy because we learned this in PHYS 25 class yesterday. Knowing something then solving for it is much more helpful for me!

I am happy with my models because it describes my observations well. Although I think I will measure my  $R$  and  $C$  better next time, rather than reading off from the instruments. I also think a PCB should be better than a breadboard to reduce stray capacitance and inductance. If I had higher resolution oscilloscopes, I would've had better results. I might want to use triggers next time so it is precisely triggered at the moment of the step transition.

End: 13:10 PM

# 20bloomw78

March 4, 2025

## 1 Figure 1

```
[87]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

# --- Load the data (assumes CSV has columns: 'Time', 'Input', 'Output')
data = pd.read_csv('20BLoolw7.csv', header=None)
data.columns = ['Time', 'Input', 'Output']

# Convert columns to numeric (forcing any non-numeric values to NaN)
data['Time'] = pd.to_numeric(data['Time'], errors='coerce')
data['Input'] = pd.to_numeric(data['Input'], errors='coerce')
data['Output'] = pd.to_numeric(data['Output'], errors='coerce')

# If your CSV is in seconds, use time as is. If in microseconds, multiply by
↳ 1e-6.
time_sec = data['Time'].values # in seconds (adjust if needed)
input_voltage = data['Input'].values
output_voltage = data['Output'].values

# --- Plot Input and Output Signals to Identify the Natural Response Region ---
plt.figure(figsize=(10, 6))
plt.plot(time_sec, input_voltage, label='Input (Square Wave)')
plt.plot(time_sec, output_voltage, label='Output (Capacitor Voltage)')
plt.xlabel('Time (s)')
plt.ylabel('Voltage (V)')
plt.title('Input and Output Signals')
plt.legend()
plt.grid(True)
plt.show()

# --- Compute the Voltage Difference ---
voltage_diff = input_voltage - output_voltage

# (Optional) Plot the voltage difference to see where it's positive
```

```

plt.figure(figsize=(10, 4))
plt.plot(time_sec, voltage_diff, 'b.-')
plt.xlabel('Time (s)')
plt.ylabel('Voltage Diff (V_in - V_out)')
plt.title('Voltage Difference vs. Time')
plt.grid(True)
plt.show()

# -----
# STEP 1: Identify the approximate time the natural response starts.
#         For example, from my plot, you might see the input goes low
#         at t ~ 0.0056 s. We'll call this 'step_time'.
# -----
step_time = 0.0005 # 0.0005 s = 500 μs (adjust as needed)

# -----
# STEP 2: Restrict to a short window after 'step_time' for the natural decay.
#         For instance, let's look 0 to 100 μs after step_time.
# -----
t_min = step_time
t_max = step_time + 100e-6 # 100 μs after step_time

mask_time = (time_sec >= t_min) & (time_sec <= t_max)
mask_diff = (voltage_diff > 0)

# Combine the masks
mask = mask_time & mask_diff

time_sec_valid = time_sec[mask]
voltage_diff_valid = voltage_diff[mask]

# Check if we have valid points
if len(time_sec_valid) == 0:
    raise ValueError("No valid data points found in the specified window.␣
↳Adjust step_time or t_max.")

# -----
# STEP 3: Take the natural log of the voltage difference
# -----
ln_diff_valid = np.log(voltage_diff_valid)

# (Optional) Shift time so that the natural response starts at t=0
# This step makes the fit more intuitive (t=0 => start of discharge).
time_sec_valid_shifted = time_sec_valid - step_time

# --- Plot the Logarithm of the Voltage Difference ---
plt.figure(figsize=(10, 6))

```

```

plt.plot(time_sec_valid_shifted, ln_diff_valid, 'bo', label='Data (ln(Input -
↳Output))')
plt.xlabel('Time after Step (s)')
plt.ylabel('ln(Voltage Difference)')
plt.title('Logarithm of Voltage Difference vs Time (Natural Response)')
plt.grid(True)
plt.legend()
plt.show()

# --- Define a Linear Function for Fitting: ln_diff = ln(V0) - t/
def linear_func(t, intercept, slope):
    return intercept + slope * t

# --- Fit the Linear Model to the Valid Data ---
popt, pcov = curve_fit(linear_func, time_sec_valid_shifted, ln_diff_valid)
intercept_fit, slope_fit = popt
intercept_err, slope_err = np.sqrt(np.diag(pcov))

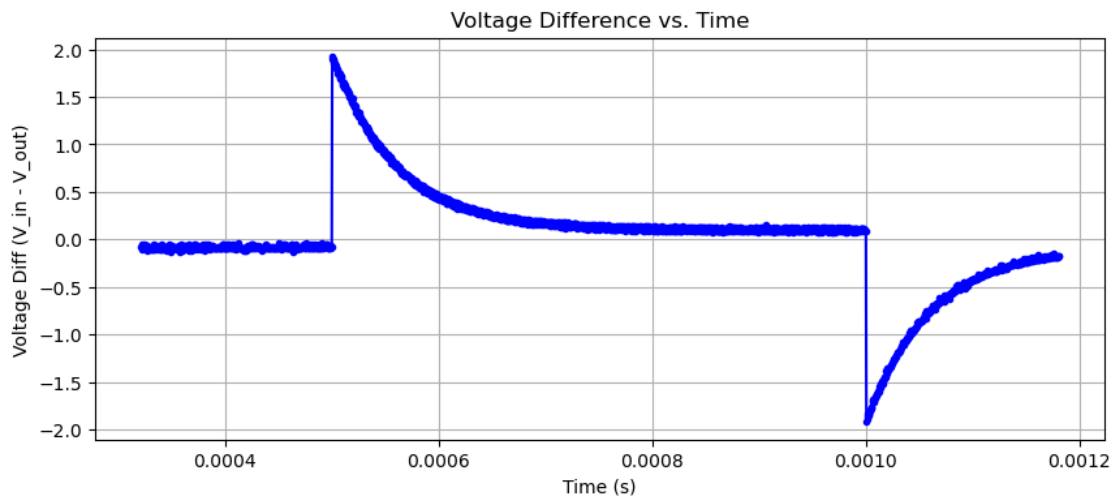
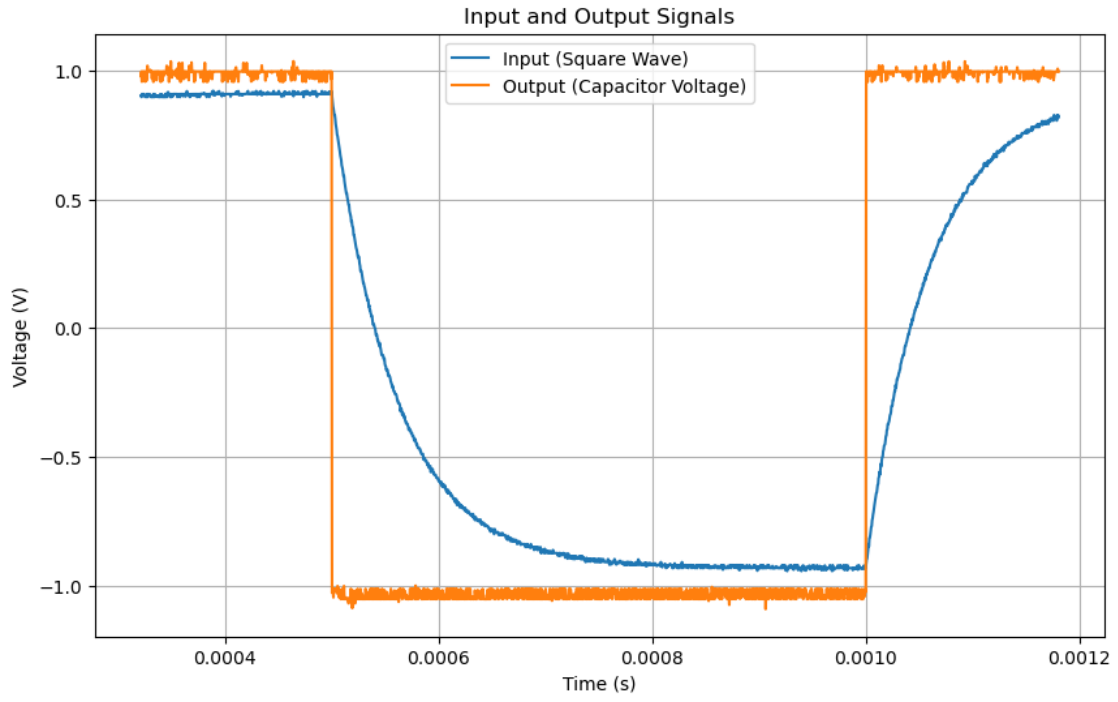
# --- Plot the Best-Fit Line ---
t_fit = np.linspace(time_sec_valid_shifted.min(), time_sec_valid_shifted.max(),
↳200)
ln_fit = linear_func(t_fit, intercept_fit, slope_fit)

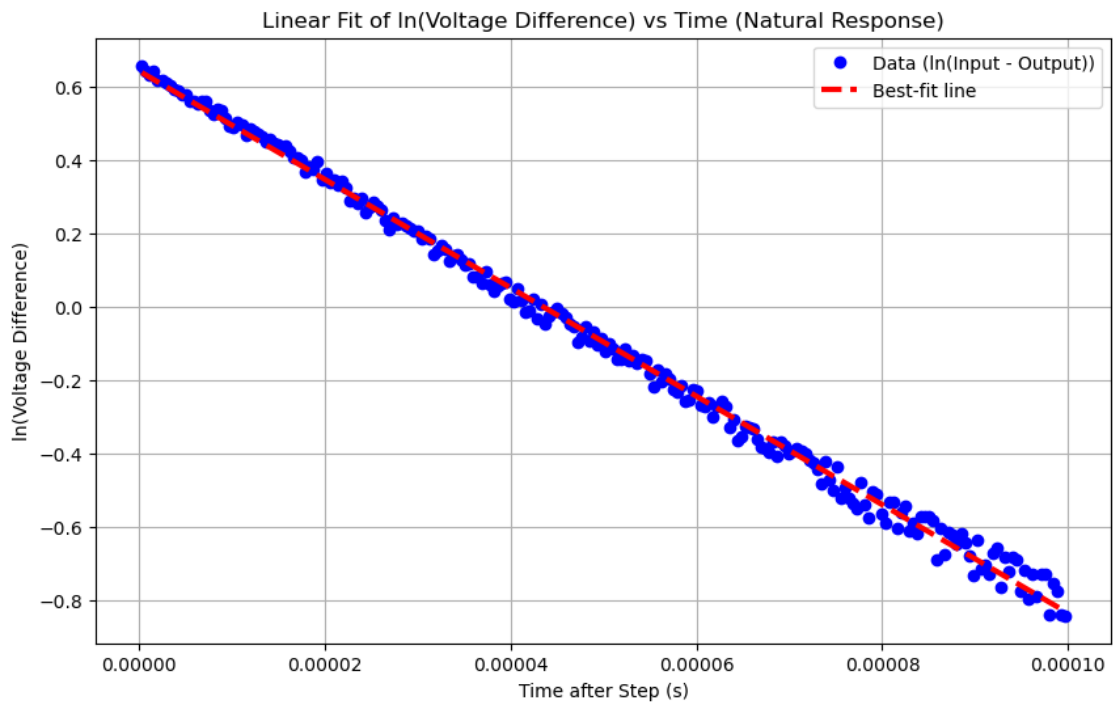
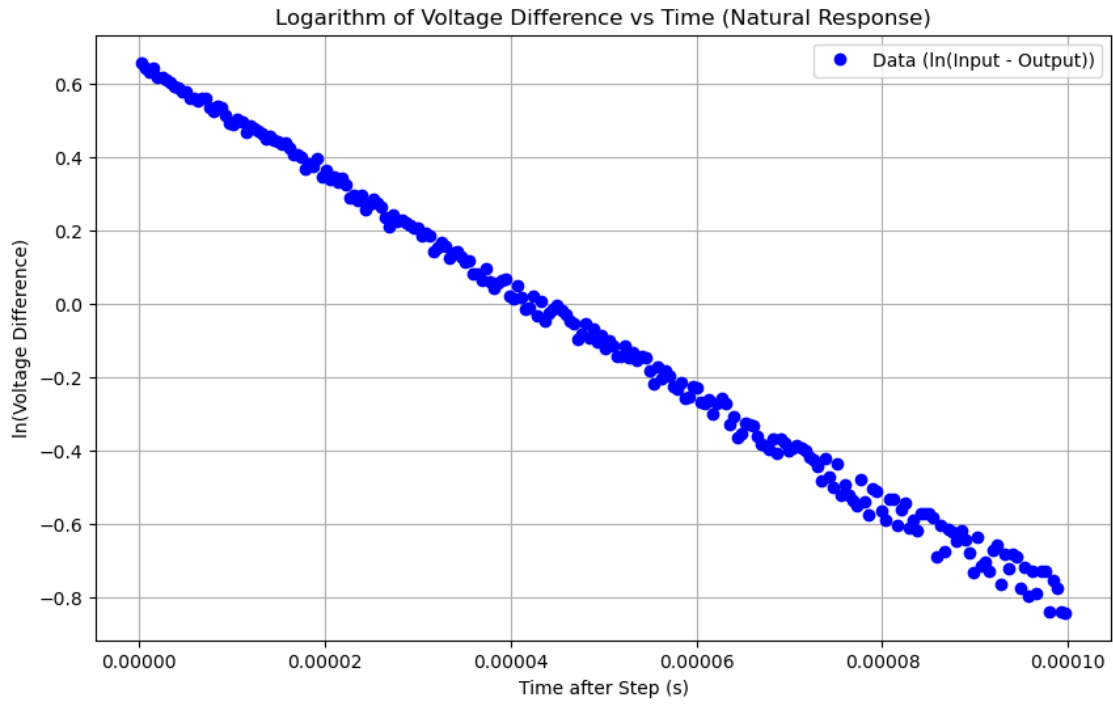
plt.figure(figsize=(10, 6))
plt.plot(time_sec_valid_shifted, ln_diff_valid, 'bo', label='Data (ln(Input -
↳Output))')
plt.plot(t_fit, ln_fit, 'r--', linewidth=3, label='Best-fit line')
plt.xlabel('Time after Step (s)')
plt.ylabel('ln(Voltage Difference)')
plt.title('Linear Fit of ln(Voltage Difference) vs Time (Natural Response)')
plt.grid(True)
plt.legend()
plt.show()

# --- Calculate the Time Constant: slope = -1/
tau_measured = -1.0 / slope_fit
tau_uncertainty = tau_measured * (slope_err / abs(slope_fit))

print(f"Intercept: {intercept_fit:.2e} ± {intercept_err:.2e}")
print(f"Slope:      {slope_fit:.2e} ± {slope_err:.2e}")
print(f"Measured time constant (tau): {tau_measured:.2e} s ± {tau_uncertainty:.
↳2e} s")

```





Intercept:  $6.43e-01 \pm 3.32e-03$   
 Slope:  $-1.48e+04 \pm 5.75e+01$

Measured time constant ( $\tau$ ):  $6.76e-05$  s  $\pm$   $2.63e-07$  s

## 2 Figure 2

```
[99]: import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

# Given information (in volts):
Vi_step = -0.94125 # Initial voltage of step response (-941.25 mV)
Vf_step = 0.91575 # Final voltage of step response (+915.75 mV)
tau_true = 58e-6 # True time constant = 58 microseconds

# The total voltage change for the step is:
V0 = Vf_step - Vi_step # Should be about 1.857 V

# Create a time vector for the step response from 0 to 500 microseconds.
t_step = np.linspace(0, 500e-6, 500) # time in seconds

# Define the step response function:
#  $V(t) = V_f + (V_i - V_f) \exp(-t/\tau)$ 
def step_response(t, tau):
    return Vf_step + (Vi_step - Vf_step) * np.exp(-t / tau)

# Compute the step response using the true tau (for simulation)
V_step = step_response(t_step, tau_true)

# For the linearization, we use the fact that for an RC charging process:
#  $V_f - V(t) = V_0 \exp(-t/\tau)$ 
# Taking the natural logarithm gives:
#  $\ln(V_f - V(t)) = \ln(V_0) - t/\tau$ 
# We'll compute  $\ln(V_f - V(t))$  from our simulated data.
diff_step = Vf_step - V_step
ln_diff_step = np.log(diff_step)

# Now we perform a linear fit to  $\ln\_diff\_step$  vs.  $t$ 
def linear_func(t, intercept, slope):
    return intercept + slope * t

popt, pcov = curve_fit(linear_func, t_step, ln_diff_step)
intercept_fit, slope_fit = popt
intercept_err, slope_err = np.sqrt(np.diag(pcov))

# Calculate the time constant from the slope.
# The theoretical relation is:  $slope = -1/\tau$ , so:
tau_fitted = -1.0 / slope_fit
tau_fitted_err = tau_fitted * (slope_err / abs(slope_fit))
```



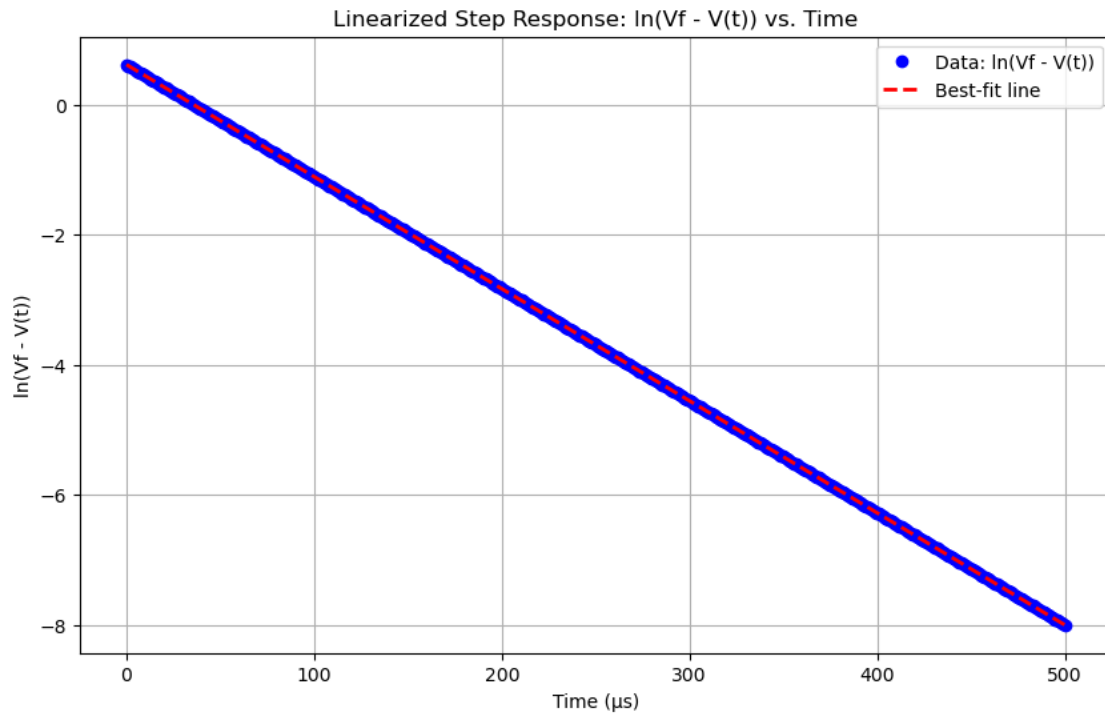
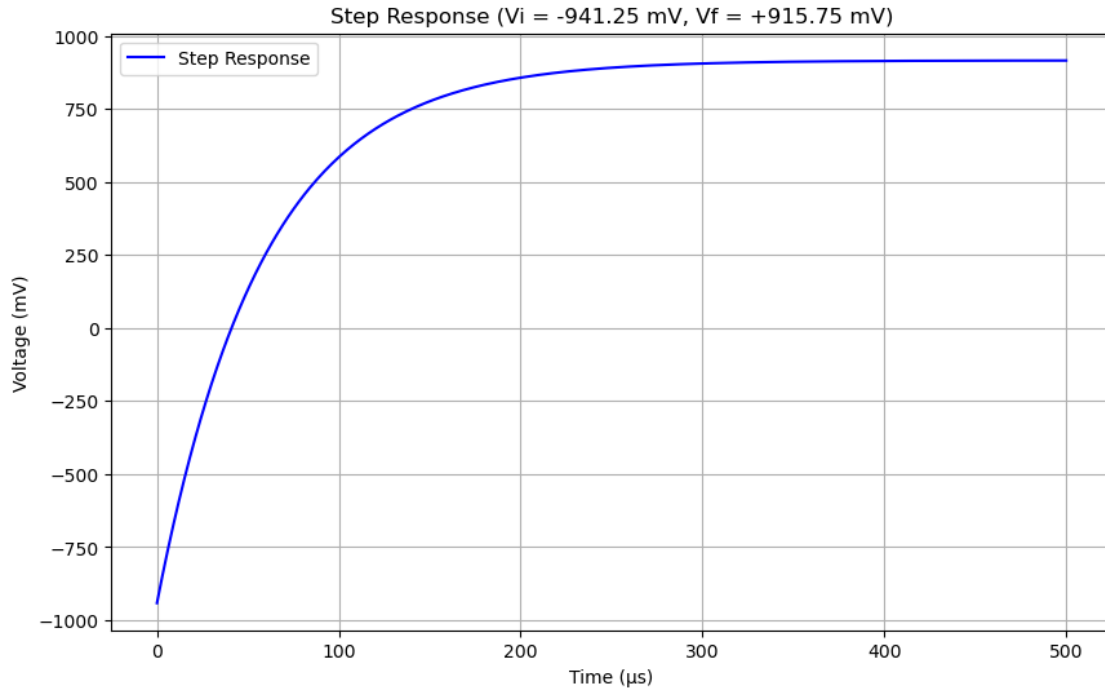
```

# Plot the original step response
plt.figure(figsize=(10,6))
plt.plot(t_step * 1e6, V_step * 1e3, 'b-', label='Step Response')
plt.xlabel('Time ( $\mu$ s)')
plt.ylabel('Voltage (mV)')
plt.title('Step Response ( $V_i = -941.25$  mV,  $V_f = +915.75$  mV)')
plt.grid(True)
plt.legend()
plt.show()

# Plot the logarithm of the voltage difference along with the best-fit line
plt.figure(figsize=(10,6))
plt.plot(t_step * 1e6, ln_diff_step, 'bo', label='Data:  $\ln(V_f - V(t))$ ')
t_fit = np.linspace(t_step.min(), t_step.max(), 200)
plt.plot(t_fit * 1e6, linear_func(t_fit, intercept_fit, slope_fit),
         'r--', linewidth=2, label='Best-fit line')
plt.xlabel('Time ( $\mu$ s)')
plt.ylabel('ln( $V_f - V(t)$ )')
plt.title('Linearized Step Response:  $\ln(V_f - V(t))$  vs. Time')
plt.grid(True)
plt.legend()
plt.show()

# Print the linear fit parameters and the computed tau
print("Fitted linear parameters:")
print("  Y-intercept: {:.2e}  $\pm$  {:.2e}".format(intercept_fit, intercept_err))
print("  Slope:      {:.2e}  $\pm$  {:.2e}".format(slope_fit, slope_err))
print("Calculated time constant ( $\tau$ ): {:.2e} s  $\pm$  {:.2e} s".format(tau_fitted,
     $\tau$ _fitted_err))

```



Fitted linear parameters:  
 Y-intercept:  $6.19\text{e-}01 \pm 1.76\text{e-}15$

Slope:  $-1.72e+04 \pm 6.08e-12$   
Calculated time constant ( $\tau$ ):  $5.80e-05 \text{ s} \pm 2.04e-20 \text{ s}$

### 3 Figure 3

```
[95]: import numpy as np
import matplotlib.pyplot as plt

# Given information (in volts):
# Step response: initial = -941.25 mV, final = +915.75 mV
# Natural response: initial = +915.75 mV, final = -941.25 mV
# Both have a measured tau = 58 microseconds

Vi_step = -0.94125 # -941.25 mV in volts
Vf_step = 0.91575 # 915.75 mV in volts
Vi_nat = 0.91575 # 915.75 mV in volts
Vf_nat = -0.94125 # -941.25 mV in volts
tau = 58e-6 # 58 microseconds in seconds

# Time arrays for plotting:
# 1) Step response from t=0 to 500 microseconds
# 2) Natural response from t=500 microseconds to 1000 microseconds
t_step = np.linspace(0, 500e-6, 500) # 0 + 500 s
t_nat = np.linspace(500e-6, 1000e-6, 500) # 500 + 800 s

# Define the exponential step response:
# V_step(t) = Vf_step + (Vi_step - Vf_step)*exp(-t/tau)
def step_response(t):
    return Vf_step + (Vi_step - Vf_step)*np.exp(-t / tau)

# Define the natural (discharge) response:
# V_nat(t) = Vf_nat + (Vi_nat - Vf_nat)*exp(-(t - t0)/tau), for t >= t0=500 s
def natural_response(t):
    t0 = 500e-6 # The discharge starts at 500 s
    return Vf_nat + (Vi_nat - Vf_nat)*np.exp(-(t - t0) / tau)

# Compute the voltages
V_step = step_response(t_step)
V_nat = natural_response(t_nat)

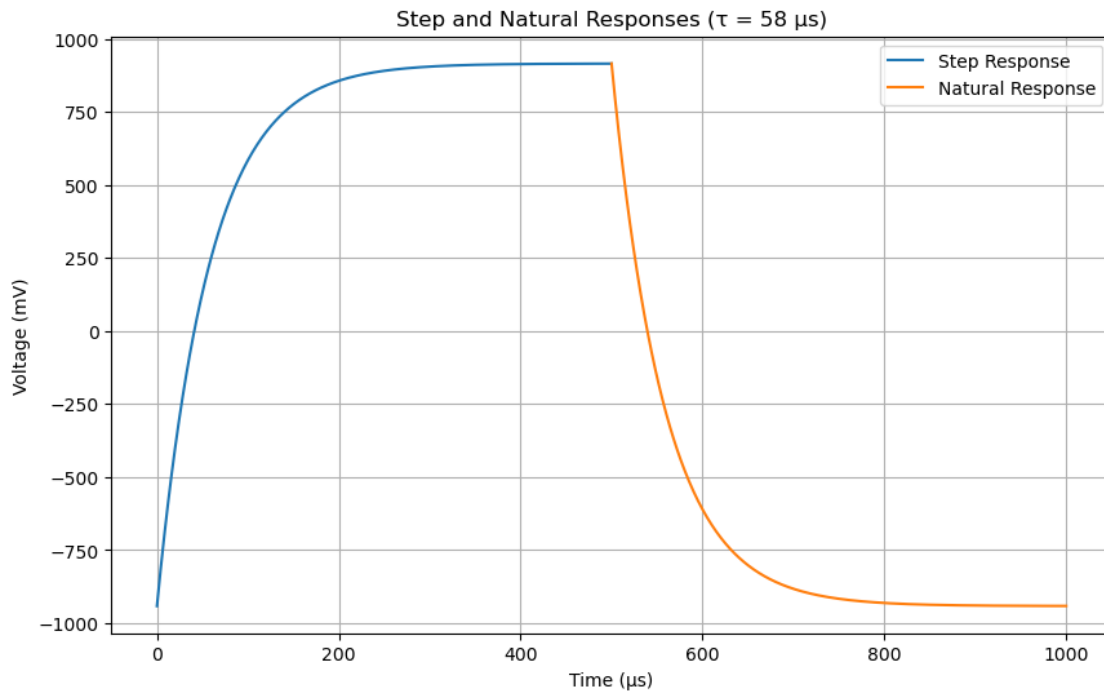
# Plot both on one figure
plt.figure(figsize=(10,6))

# Convert time to microseconds and voltage to millivolts for readability
plt.plot(t_step * 1e6, V_step * 1e3, label='Step Response')
plt.plot(t_nat * 1e6, V_nat * 1e3, label='Natural Response')
```

```

plt.xlabel('Time (μs)')
plt.ylabel('Voltage (mV)')
plt.title('Step and Natural Responses (τ = 58 μs)')
plt.grid(True)
plt.legend()
plt.show()

```



#### 4 Figure 4

```

[35]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

# --- Load and Prepare Data ---
# Load the CSV file. Since your CSV appears to have 3 columns, we handle that.
data = pd.read_csv('songgun8.csv', header=None)

# Determine the number of columns and assign names accordingly.
if data.shape[1] == 2:
    data.columns = ['Time', 'Voltage']
elif data.shape[1] == 3:
    data.columns = ['Time', 'Voltage', 'Extra'] # 'Extra' column is ignored in
↳ this analysis.

```

```

else:
    raise ValueError("Unexpected number of columns in CSV file.")

# Convert 'Time' and 'Voltage' columns to numeric values
data['Time'] = pd.to_numeric(data['Time'], errors='coerce')
data['Voltage'] = pd.to_numeric(data['Voltage'], errors='coerce')

# Convert time from microseconds to seconds
time = data['Time'].values          # in microseconds
time_sec = time * 1e-6             # in seconds

# Convert voltage from mV to Volts
voltage = data['Voltage'].values * 1e-3 # in Volts

# --- Plot the Raw Damped Oscillation Signal ---
plt.figure(figsize=(10, 6))
plt.plot(time_sec, voltage, label='Inductor Voltage', color='blue')
plt.xlabel('Time (s)')
plt.ylabel('Voltage (V)')
plt.title('Damped Oscillator: Inductor Voltage vs Time')
plt.legend()
plt.grid(True)
plt.show()

# --- Estimating the Time Constant from the Envelope ---
# Given envelope peak points from your measurement:
# (5.1 s, 263.25 mV) and (12.0 s, 101.25 mV)
t1_us, t2_us = 5.1, 12.0          # in microseconds
V1_mV, V2_mV = 263.25, 101.25    # in millivolts

# Convert these to SI units:
t1 = t1_us * 1e-6                # seconds
t2 = t2_us * 1e-6                # seconds
V1 = V1_mV * 1e-3                # Volts
V2 = V2_mV * 1e-3                # Volts

# For an exponential envelope:  $V(t) = A * \exp(-t/)$ 
# At  $t_1$ :  $V_1 = A * \exp(-t_1/)$  and at  $t_2$ :  $V_2 = A * \exp(-t_2/)$ 
# Taking the ratio gives:  $V_1/V_2 = \exp((t_2-t_1)/)$ 
# Solving for :
tau_est = (t2 - t1) / np.log(V1/V2)
print("Estimated time constant (tau) from envelope peaks: {:.2e} s".
      ↪format(tau_est))

# --- Plotting the Estimated Exponential Envelope ---
# Reconstruct the envelope using the exponential decay:
# Using the value at  $t_1$ , solve for  $A$ :  $A = V_1 * \exp(t_1/)$ 

```

```

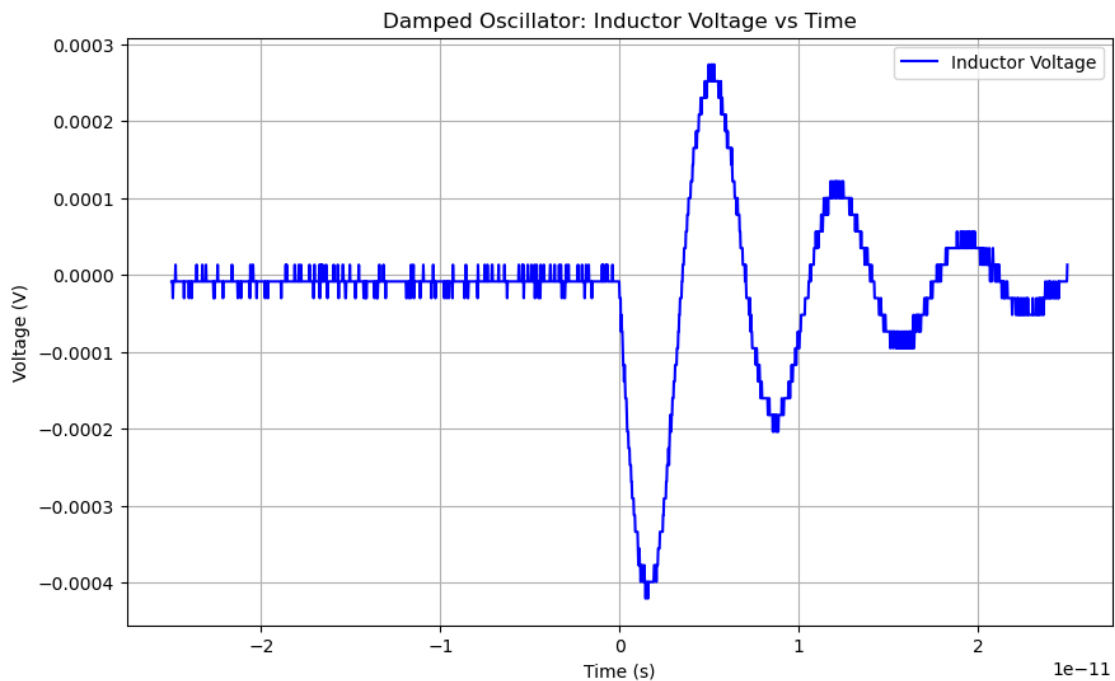
A = V1 * np.exp(t1 / tau_est)

# Define the envelope function
def envelope(t):
    return A * np.exp(-t / tau_est)

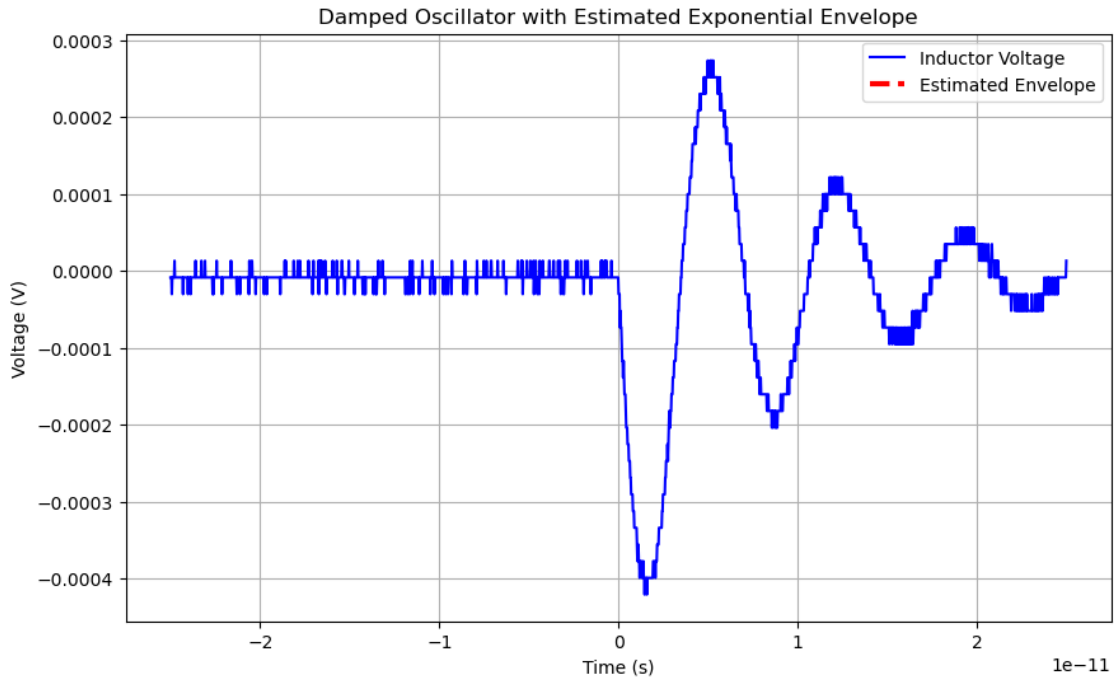
# Generate a fine time array for plotting the envelope
t_envelope = np.linspace(time_sec.min(), time_sec.max(), 500)
envelope_values = envelope(t_envelope)

plt.figure(figsize=(10, 6))
plt.plot(time_sec, voltage, label='Inductor Voltage', color='blue')
plt.plot(t_envelope, envelope_values, color='red', linestyle='--', linewidth=3,
        label='Estimated Envelope')
plt.xlabel('Time (s)')
plt.ylabel('Voltage (V)')
plt.title('Damped Oscillator with Estimated Exponential Envelope')
plt.legend()
plt.grid(True)
plt.show()

```



Estimated time constant ( $\tau$ ) from envelope peaks:  $7.22 \times 10^{-6}$  s



5 Figure 5

```
[113]: import numpy as np
import matplotlib.pyplot as plt

# Measured data for the envelope (in SI units):
t1 = 5.1e-6          # 5.1 microseconds
V1 = 263.25e-3      # 263.25 mV in volts
t2 = 12.0e-6        # 12.0 microseconds
V2 = 101.25e-3      # 101.25 mV in volts

# Calculate the decay time constant tau using:
# tau = (t2 - t1) / ln(V1/V2)
tau = (t2 - t1) / np.log(V1/V2)
print("Calculated tau =", tau, "seconds") # Expected ~7.22e-6 s

# Calculate V0 from the first point: V1 = V0 * exp(-t1/tau)
V0 = V1 * np.exp(t1/tau)
print("Calculated V0 =", V0, "volts")

# Create a time array for the envelope, e.g., 0 to 20 microseconds
t = np.linspace(0, 20e-6, 200)

# Compute the ideal envelope:
```

```

V_ideal = V0 * np.exp(-t/tau)

# --- Plot 1: Exponential Decay Envelope ---
plt.figure(figsize=(10,6))
plt.plot(t*1e6, V_ideal*1e3, 'r-', linewidth=2, label='Ideal Exponential Decay')
plt.plot(t1*1e6, V1*1e3, 'bo', markersize=8, label='Measured Peak 1')
plt.plot(t2*1e6, V2*1e3, 'go', markersize=8, label='Measured Peak 2')
plt.xlabel('Time (µs)')
plt.ylabel('Voltage (mV)')
plt.title('Exponential Decay Envelope')
plt.legend()
plt.grid(True)
plt.show()

# --- Plot 2: Linearized Plot ---
# Taking the natural logarithm:
ln_V = np.log(V_ideal) # ln(V(t)) = ln(V0) - t/tau

plt.figure(figsize=(10,6))
plt.plot(t*1e6, ln_V, 'b-', linewidth=2, label=r'$\ln(V(t)) = \ln(V_0) - t/\tau$')
plt.xlabel('Time (µs)')
plt.ylabel('ln(Voltage)')
plt.title('Linearized Exponential Decay (slope = -1/tau)')
plt.grid(True)
plt.legend()
plt.show()

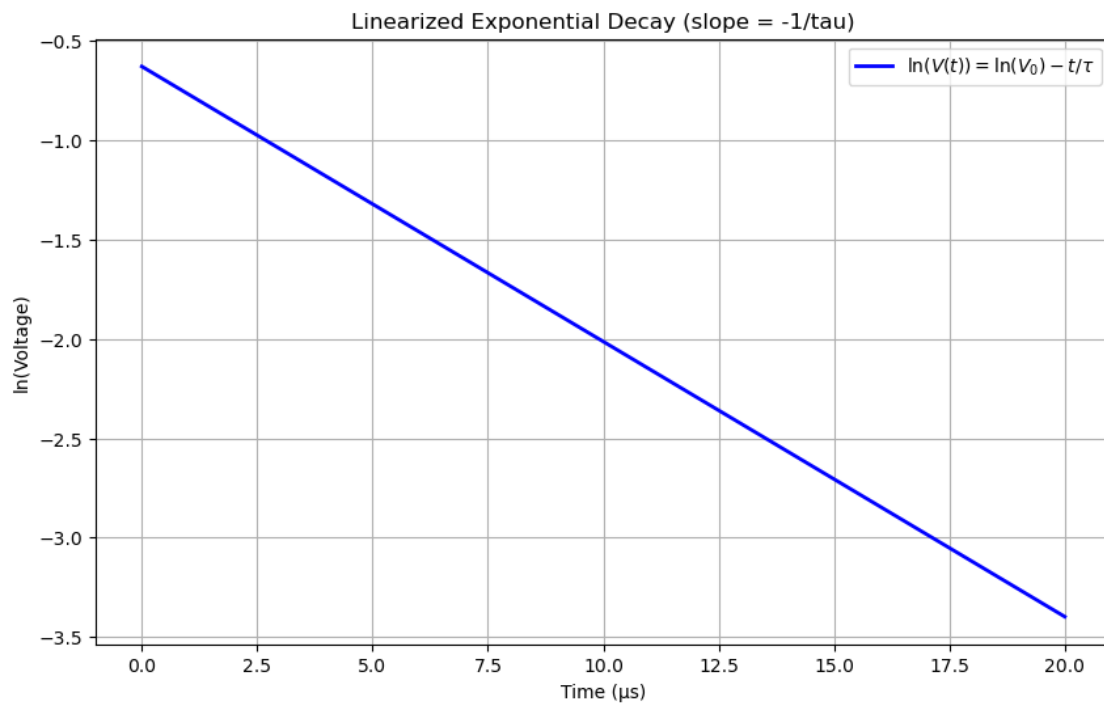
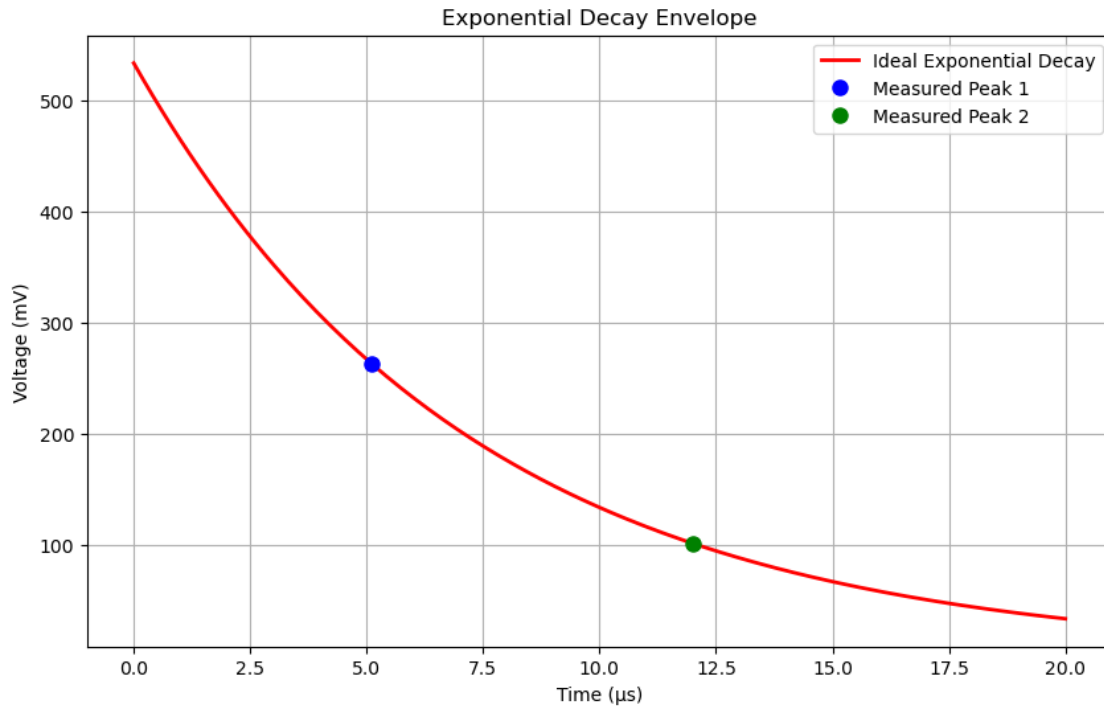
# Print the expected linear parameters:
intercept_theoretical = np.log(V0)
slope_theoretical = -1.0 / tau
print("Theoretical intercept (ln(V0)) =", intercept_theoretical)
print("Theoretical slope (-1/tau) =", slope_theoretical)

```

Calculated tau = 7.221263581831692e-06 seconds

Calculated V0 = 0.5334427425578737 volts





Theoretical intercept ( $\ln(V_0)$ ) = -0.6284035381651645  
 Theoretical slope ( $-1/\tau$ ) = -138479.91956919365